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Backscattering Measurement Method for Sound Field using Pulsed Laser

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Summary
This paper presents a sound field measurement method utilizing light scattering. Since the intensity of scattered light is proportional to density of scatterers, the fluctuation of air density, which is corresponding to sound pressure, can be acquired by observing the light scattered by air particles. The light scattered in the sound field, therefore, includes the sound information in its intensity. The measurement apparatus is composed of a pulsed laser, a telescope and a photo-multiplier tube (PMT). The backscattered light is collected by the telescope and detected by the PMT. Using the pulsed laser and detecting the backscattered light enable to acquire sound quantity of an arbitrary point on optical light path by controlling signal acquisition timing. The results of the fundamental experiments confirm a validity of this method.

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1. Introduction

Microphones are usually used for sound field measurement. However a concern is microphones disturb the field. In addition, it is difficult to arrange the microphones within sufficiently small intervals for measuring spatial behavior of sound waves. Therefore, non-intrusive sound field measurement methods with optical instruments have been studied recently. The sound field measurement method with laser Doppler vibrometer (LDV) has been proposed [1–6]. There are studies on sound visualization by the schlieren method [7] and sound measurement by the optic wave microphones [8].

These conventional methods measure integrated value of acoustical quantity along optical path because of detecting reflected or transmitted light. These methods, therefore, are difficult to measure the sound quantity of one point directly.

In this study, we propose the optical sound measurement method that can measure non-integrated sound quantity with no disturbance to the field. The technique of LIDAR (LIght Detection And Ranging), which is observation system of weather condition using a pulsed laser and a telescope [9], is applied to the sound field measurement method that captures scattered light intensity proportional to density of scatterers. Pulsed light is emitted to the sound field and the scattered light by air particles in the field are detected. The scattered light has the sound information in its intensity of the points where the particles present. The technique allows us to measure sound of an arbitrary on the optical path. This paper describes theories and fundamental experiments of the proposed method.

2. Light scattering theory

2.1. Light Scattering

Light scattering, that is the incident light diffused by particles, is categorized into two types: *elastic scattering* and *inelastic scattering*. Elastic scattering is the scattering that the sum of kinetic and internal energy does not change by the scattering. On the other hand, inelastic scattering is the energy changes after the scattering.

Scattering by sound, elastic scattering, depends on a ratio of a wavelength of incident light and a radius of scatterer. The *size parameter* a, which is often used for parameterizing the radius of scatterers, is represented as

$$a = \frac{2\pi d}{\lambda}, \tag{1}$$

where d is the radius of the scatterer, λ is the wavelength of the light. If a is much smaller than 1, the scattering is called *Rayleigh scattering* that is caused by nitrogen and oxygen molecules in the atmosphere. If a is nearly equal to 1, that is called *Mie scattering* caused by water drops and dust.

2.2. Mie Scattering

Gustov Mie solved the equation for a spherical and homogenous particle and a monochromatic plane wave in 1908 [11]. A form of a *scattering cross section* σ, which is defined as the number of particles scattered when single photon is entered unit section and unit time, is represented as follows:

$$\sigma = \frac{\lambda^2}{2\pi}\sum_{n=1}^{\infty}(2n+1)\left[|a_n|^2 + |b_n|^2\right], \tag{2}$$

where a_n and b_n are

$$a_n = \frac{\psi_n(a)\psi_n'(ma) - m\psi_n(ma)\psi_n'(a)}{\zeta_n(a)\psi_n'(ma) - m\psi_n(ma)\zeta_n'(a)}, \tag{3}$$

$$b_n = \frac{m\psi_n(a)\psi_n'(ma) - \psi_n(ma)\psi_n'(a)}{m\zeta_n(a)\psi_n'(ma) - \psi_n(ma)\zeta_n'(a)}, \tag{4}$$

m is the relative refractive index, ψ_n is the Ricatti-Bessel function, and ζ_n is the Hankel function of the first kind [12].

2.3. Rayleigh Scattering

Since the scattering by the particle whose diameter is much smaller than the wavelength of the incident light is considered as a radiation from a single dipole oscillated by an applied electro-magnetic field, the scattering equation alters a simple form by an appropriate approximation [13]. When unpolarized light enters a homogenous particle, the scattering intensity at distance r is

$$I(\theta) = \frac{k^4 d^6}{2r^2}\left|\frac{m^2-1}{m^2+1}\right|^2 (1+\cos^2\theta), \tag{5}$$

where k is the wave number. The intensity of Rayleigh scattering is proportional to the 6th power of the diameter of the particle and the minus 4th power of the wavelength of the incident light. The scattering cross section is written as

$$\sigma = \frac{8\pi}{3}k^4 d^6\left|\frac{m^2-1}{m^2+2}\right|^2. \tag{6}$$

2.4. Sound pressure acquisition from fluctuation of scattering intensity

A *scattering coefficient* β, which is the ratio of intensity of incident light and scattered light, is

$$\beta(\theta) = N\frac{d\sigma(\theta)}{d\Omega} = \frac{I(\theta)}{I_0}, \tag{7}$$

where θ is the angle formed by the incident light and the scattered light, N is the number density which is a number of scatterers per unit volume, and $d\sigma/d\Omega$ is the *differential scattering cross section*, I is the intensity of the scattered light, and I_0 is the intensity of the incident light. The number density is

$$N = M\rho, \tag{8}$$

where M is molecular weight of the scatterer and ρ is density of scatterer. Therefore, by substituting Eq.(8) into Eq.(7), the scattered light intensity is

$$I(\theta) = I_0 M\frac{d\sigma(\theta)}{d\Omega}\rho. \tag{9}$$

The differential scattering cross section is *scattering cross section* for angle θ. In other words, the scattering cross section, which is depended on the wavelength of the incident light, the diameter of the scatterer and the reflective index, is an integrated value of the differential scattering cross section with respect to angle. When we consider an actual measurement system, the wavelength of the incident light and the diameter of the scatterer can be assumed constant. If the light source and the receiving optics do not move, the angle formed by the incident light and the optics is also constant. This means, from Eq.(9), the intensity of the scattered light is proportional to the scatterer density. Therefore, we can get sound pressure from intensity fluctuation of the scattered light.

3. Sound measurement system

3.1. Non-integrated acoustical quantity measurement with pulsed light

An outline of scattering using a continuous wave (CW) laser and a pulsed laser is illustrated in Fig. 1. If we use the CW laser as light source, detected value is the sum of scattered light from all points on the optical path of the laser beam because the scattering by air particles occurs wherever light and air are. On the other hand, if we use the pulsed laser, the value is not integrated because the scattering area is where the emitted pulsed light presents.

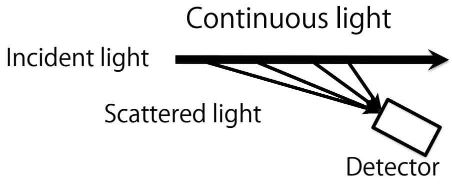
Although the scattering area is limited by using the pulsed light, it is necessary to consider effect of sound waves on the optical path to get sound quantity of specific volume. If the transmitted light is affected by the sound, since the detected light contains the sound quantity at not only a scattering point but also all points where the detected light passed, we cannot acquire non-integrated quantity. Then, we consider intensity change of the transmitted light caused by the sound waves where the light passing through.

An *atmospheric transmissivity*, which is a ratio of intensity of the incident light and that of the transmitted light when the light propagates distance R, is

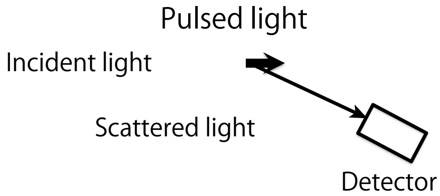
$$T(R) = \exp\left(-\int_0^R \alpha\, dr\right), \tag{10}$$

where α is called an *extinction coefficient*, which is the sum of scattering coefficients and absorption coefficients. The air particles are composed of molecules and aerosol. Thus the extinction coefficient α is written as

$$\alpha = \alpha_{\mathrm{scat,mol}} + \alpha_{\mathrm{scat,aer}} + \alpha_{\mathrm{abs,mol}} + \alpha_{\mathrm{abs,aer}}, \tag{11}$$



(a) Continuous light



(b) Pulsed light

Figure 1. Sketch of an outline of the difference of scattering behavior between (a) continuous light and (b) pulsed light when scatterers are air particles. Since the scattering occurs at the points where the light exists, the continuous light is scattered at all points on the path. On the other hand, the pulsed light is scattered at only limited volume determined by the laser beam diameter and the pulse width.

where $\alpha_{\text{scat},\cdot}$ is the scattering coefficients, $\alpha_{\text{abs},\cdot}$ is the absorption coefficients, $\alpha_{\cdot,\text{mol}}$ is the coefficients by molecules and $\alpha_{\cdot,\text{aer}}$ is the coefficients by aerosol. Although the extinction coefficient is also changed by wind and temperature fluctuation, we can ignore these effects because the frequencies are low enough to distinguish from audible sound frequencies.

When the scattering coefficient changes by the sound wave on the optical path except for the scattering point, the rate of variation in the atmospheric transmissivity $T'(R)$ is

$$T'(R) = \exp(-\Delta\alpha_s), \tag{12}$$

where $\Delta\alpha_s$ is variation in the scattering coefficient in unit length. Variation in the absorption coefficient is regarded as zero. On the other hand, when the same sound wave is at the scattering point, rate of variation in a scattering coefficient β' is

$$\beta' = \frac{\beta + \Delta\beta}{\beta}, \tag{13}$$

where $\Delta\beta$ is variation in the scattering coefficient. If changes in air pressure at the scattering point is 1 Pa, the ratio of $\Delta\beta$ and β is 10^{-5}. Thus

$$\beta' = 1 + 10^{-5}. \tag{14}$$

A scattering coefficient of US standard atmosphere [14] is

$$\alpha_s = 7.5 \times 10^{-5} \ [\text{m}^{-1}], \tag{15}$$

when the wavelength of the incident light is 350 nm. The ratio of fluctuation of the received light by the sound wave at the scattering point and the points except for the scattering point is

$$\left| \frac{1 - \beta'}{1 - T'(R)} \right| = 82.5 \ [\text{dB}], \tag{16}$$

which indicates that the fluctuation of the received signal by $T'(R)$ is negligible so that $1 - T'(R)$ is much smaller than $1 - \beta'$. In other words, the detected light is regarded not to include sound quantity other than the scattering point with good approximation.

3.2. Sound field measurement based on LIDAR system

LIDAR is a weather observation system used in atmospheric science [9, 10]. Pulsed light is emitted to the sky and the light scattered by air particles is detected using a telescope and a photo-multiplier tube (PMT). LIDAR can measure temperature, air pressure, humidity, wind velocity and other weather quantities from intensity, spectrum and polarization of the scattered light.

The sound field measurement system applied LIDAR techniques enables to measure sound information of any points on the optical light path. Figure 2 shows a schematic of the proposed system. The scattered light by air is collected by a telescope and detected by a PMT. According to put the telescope as whose direction of a field of view is parallel to traveling direction of laser light, the time interval between the laser emission and the signal acquisition by the sample-and-hold circuit is proportional to the distance between the received optics and the measuring point. The time interval between the laser emission and the signal acquisition t_δ is

$$t_\delta = t_{\text{record}} - t_{\text{emit}}, \tag{17}$$

where t_{record} is the signal acquisition time ,and t_{emit} is the laser emission time. The distance from the optics to the measuring point R is

$$R = \frac{ct_\delta}{2}, \tag{18}$$

where c is the speed of light.

Received light power P is

$$P(R) = P_I \frac{c\tau}{2} \eta s A \frac{T(R)^2}{R^2} \beta, \tag{19}$$

where P_I is incident light power, τ is the pulse width of the laser light, η is the efficiency of the system, s is the cross section of the pulsed light, and A is the area of the primary receiver optics. The term $c\tau/2$ which is the pulse width contributed to the scattering is called *effective spatial pulse width*. The quadratic decrease of the received power respect to distance is due to the

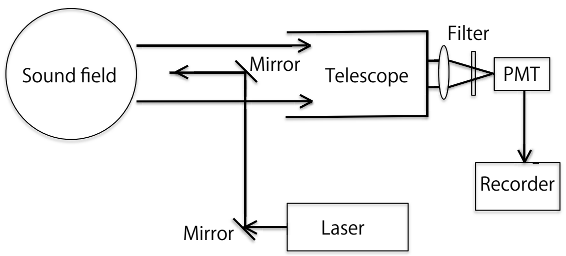


Figure 2. Schematic of the sound pressure measurement instruments using scattering by air. Pulsed light is emitted to sound field and backscattered light by air particles in the field is detected by a PMT via a telescope.

distance decay of the scatted light from the scattering point to the telescope. In Eq.(19), variables are the atmospheric transmissivity T and the scattering coefficient β. However, since β is much larger than T as described before, T is regarded as constant, and the received power P is proportional to β. Therefore, the proposed method enables to get the sound quantity of scattered point because of proportionality between the scattering coefficient and the density of scatterers.

4. Experiment

4.1. Method

Figure 3 shows the experimental arrangement. Two speakers were put 1 m away from laser light path and generated different sound. The sound of the points in front of each speaker was detected from backscattered light. The telescope whose direction of the field of view was parallel to traveling direction of the pulsed light collected the scattered light. The scattered light was converted to electric current by the PMT arranged at the focal point of the telescope. The sample-and-hold circuit was used for extracting the signal of a certain scattering point. Since the sample-and-hold circuit can hold an instantaneous voltage value, the voltage value of at the certain time after pulse emission is held by the circuit until next pulse emission. The timing of the pulse emission and the clock of the sample-and-hold circuit were controlled by the function generator. We did two measurements and set measuring points 7 m and 17 m away from the optics, respectively.

Speaker-1 put 8 m away from the optics generated sine wave of 1 kHz and Speaker-2 put 16 m away from the optics generated sine wave of 2 kHz. The sound pressure levels of generated sounds of both speakers are 90 dB. It is concerned that, if the sounds are where the optical system is, the optical system vibrates by the sound waves. This means sound quantity might be detected by vibration of the optical system. To assure that the optical system does not vibrate by sound waves, two sound sources were both 15 ms length and the analyzed signals were also 15 ms from sound generating. Since sound propagates 5.1 m during 15 ms, the sound does not reach the optical system within 15 ms.

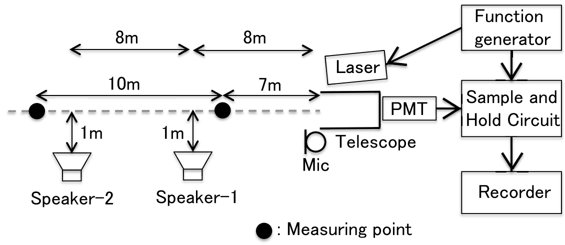


Figure 3. Schematic of the apparatus for sound pressure measurement utilizing scattering by air particles. The backscattered light is collected by a telescope and detected by a PMT. The PMT signal after certain time from laser emission is recorded.

Table I. Specification of the laser used for the experiment.

Wavelangth	355 nm
Energy	0.6 mJ
Reputation frequency	10 kHz
Pulse width	20 ns
Beam Diameter (at 10m)	1.4 mm

The specification of the laser is shown in Table I. The ultraviolet laser was used to increase the scattered light intensity because the intensity of Rayleigh scattering is inversely proportional to 4th power of the wavelength of the incident light. Since the reputation rate of the laser was 10 kHz, upper limit frequency of the measurement was 5 kHz.

4.2. Results

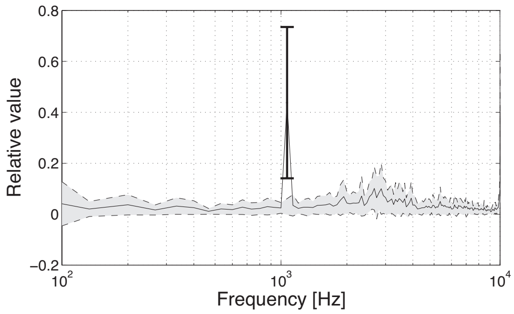
The results are shown in Figures 4 and 5. The frequency response of 10 times averaged amplitude and 68% confidence interval in frequency domain of 20 times averaged signal in time domain is illustrated. Both figures describe the relative values to the signals when the speakers generate no sound in vertical axis.

Figure 4(a) shows the 1 kHz sine wave is detected when the measuring point is 7 m away from optics. According to Fig. 4(b), however, the 2 kHz sine wave was not captured. It seems that the 2 kHz signal is buried in noise because the intensity of the received light scattered at 14 m is 5.9 times less than that of 7 m by Eq.(19).

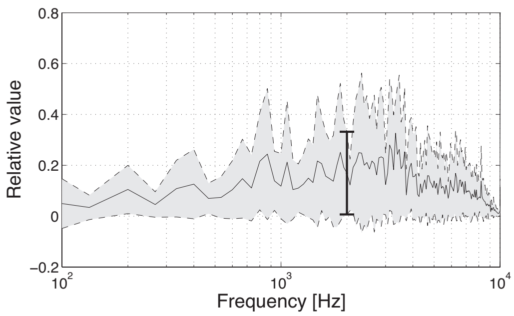
Figure 5 shows the frequency response of the sound recorded by the microphone put near the optical system. It is confirmed that the sound generated by the speakers did not affect the optics.

5. Conclusions

In this paper, the sound measurement method with light scattering is proposed. By utilizing the fact that the intensity of the scattered light is proportional to the density of the scatterers, the proposed method can measure sound quantity non-intrusively by extracting the quantity from the light scattered by air particles in sound field. Using a pulsed laser as light source enables us to measure not integrated but point value. This allows us to observe the acoustical quantity of arbitrary points without setting equipment in the measurement area. However, low SN ratio of the method due to small scattering cross section of air remains an issue. To further verify the experimental results, future work should improve the SN ratio of the method.



(a) Scattering point is 7 m away from optics.



(b) Scattering point is 17 m away from optics

Figure 4. Frequency response of 10 times averaged amplitude and 68% confidence interval in frequency domain of 20 times averaged signal in time domain when scattering points are (a) 7 m and (b) 17 m away from the optics, respectively. The solid lines are the average values and the grey regions surrounded by dotted lines shows 68% confidence intervals, however 68% confidence intervals of target frequencies are depicted by error bars.

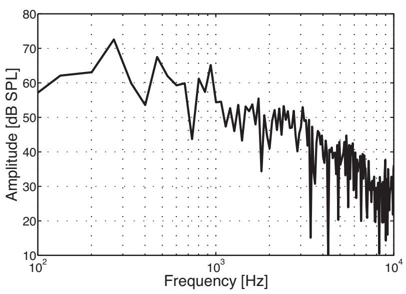


Figure 5. Frequency response of the microphone signal within 15 ms from generated sounds by speakers when speaker-1 and speaker-2 generate 1 kHz and 2 kHz sine wave respectively. The microphone is put near received optics.

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