



Optical sound field measurement and imaging using laser and high-speed camera

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ABSTRACT

Optical sound measurement, which acquires acoustical quantities by means of optical techniques, is of growing interest as an alternative method for the sound field imaging. There are two remarkable aspects of the optical sound measurement. The first is non-intrusiveness. Since the measurement is achieved by observing the light passed through the sound field, the instruments can be arranged outside the measurement field; non-contact and non-destructive measurement can be achieved. The second one is spatial resolution. Instead of building an array, expanding or scanning of light are often used for the optical imaging. Therefore, the optical imaging does not have the limitation of interval of measurement points due to the size of the instruments as microphone arrays. These two aspects make the optical method possible to image sound field with high spatial resolution and without any disturbance to the original field. In this paper, we show several methods for the optical sound imaging. Laser Doppler vibrometer can be developed as the imaging methods by scanning a narrow light beam. The two dimensional of transient field measurement can be achieved by using a high-speed camera because of single-shot. In addition some signal processing techniques introduced to optical measurement are also described.

Keywords: Acousto-optic effect, laser Doppler vibrometer (LDV), physical-model based signal processing, parallel phase-shifting interferometry (PPSI) I-INCE Classification of Subjects Number(s): 72

1. INTRODUCTION

In recent years, there has been interested in the sound field measurement and imaging methods using light [1–30]. Sound is density change of air, and the refractive index of air is modulated by the sound. So by detecting the change in the refractive index by using light which has passed through the sound field, it is possible to obtain the sound information of the optical path.

The use of light, which does not affect the sound field, becomes possible to acquire the sound information even in the places where installation of the microphone is difficult. However, there are a lot of difficulties to take an acoustical measurement by light with the signal to noise ratio (SNR) good, because the sound pressure is very small compared with the other atmospheric pressure change and the sound effects on the light, which is the acousto-optic effect, is also very small. Furthermore, some ideas for the measurement system are required in order to take the optical sound field measurement and imaging.

This paper provide a brief review of the optical sound measurement methods and the signal processing method for them that have been studied by the authors [1–14]. Firstly, we give an outline about the acousto-optic effect. Next, we show the sound field measurement method using laser Doppler vibrometer (LDV) based on that theory. The introduction of signal processing based on a physical model and optimization techniques allows the extraction of sound information from the measurement data with SNR bad. We show its application to the acoustic measurement using LDV. Finally, we describe the high-speed measurement of sound fields using parallel phase-shifting interferometry (PPSI) with a high-speed polarization camera as a most advanced optical measurement system.

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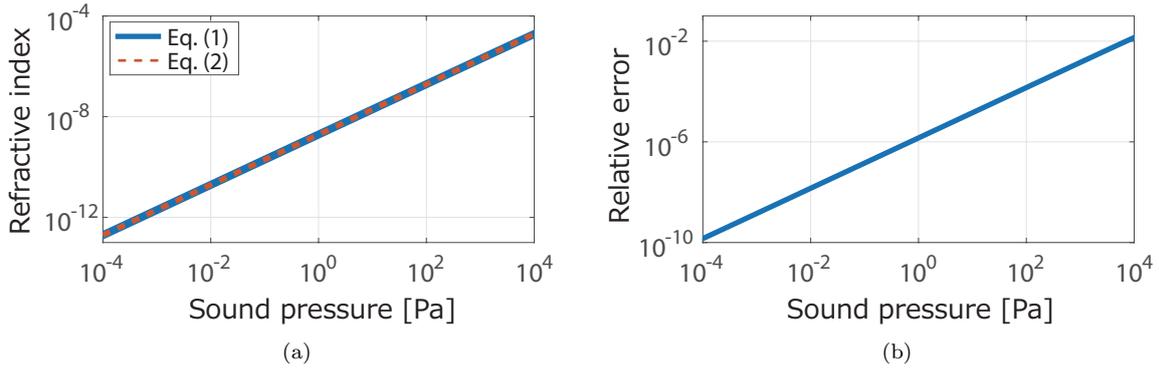


Figure 1: The relation between sound pressure and refractive index. (a) shows the variation in the refractive index of air from the static value calculated by Eq. (1) (blue line) and Eq. (2) (dashed red line). (b) shows the relative error due to the linear approximation. The constants are the following: $n_0 = 1.000279$, $\gamma = 1.41$, and $p_0 = 101325$.

2. PRINCIPLE OF OPTICAL SOUND MEASUREMENT

This section reviews the physical relation between light and audible sound in air for the optical sound measurement. Some more details of the presenting information can be found in [12, 13].

Modulation of light caused by sound is well-known as acousto-optic effect in the field of ultrasonic. That is modeled as deflection, diffraction or phase modulation. The models depend on a pressure and wavelength of sound. For a weak sound in air, the relation between sound pressure, p , and refractive index of air, n , is

$$n = (n_0 - 1) \left(1 + \frac{p}{p_0} \right)^{1/\gamma} + 1, \quad (1)$$

where n_0 and p_0 are the refractive index and pressure under static condition, respectively; γ is the specific heat ratio. When p/p_0 is much smaller than 1, a linear relation can be assumed, that is,

$$n = n_0 + \frac{n_0 - 1}{\gamma p_0} p. \quad (2)$$

Figure 1 plots the relation between the sound pressure and the refractive index calculated by Eqs. (1) and (2), and the difference of Eqs. (1) and (2), respectively. The figure indicates that the relative error of the linear approximation is less than 10^{-4} for the audible sound pressure range. Therefore, the linear approximation is appropriate for the relation between the audible sound and the refractive index of air.

According to the theory of geometrical optics, light propagates in inhomogeneous media can be represented as

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 e^{i(k \int_{\mathbf{L}(\mathbf{r})} n(\mathbf{l}, t) d\mathbf{l} - \omega t)} \quad (3)$$

$\mathbf{r} = (x, y, z)$ is the position of the three-dimensional Euclidean space; t is the time; ω is the angular frequency of light; k is the wavenumber of light; and $\mathbf{L}(\mathbf{r})$ is the optical path from laser emitting point to \mathbf{r} . By substituting Eq. (2) into Eq. (3) light propagated in the weak sound field can be written as

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 e^{i(k n_0 L_0 - \omega t)} e^{i\phi_p}, \quad (4)$$

where L_0 is the propagation distance of light and ϕ_p is the phase modulation term, which is given by

$$\phi_p(\mathbf{r}, t) = k \frac{n_0 - 1}{\gamma p_0} \int_{\mathbf{L}(\mathbf{r})} p(\mathbf{l}, t) d\mathbf{l}. \quad (5)$$

Equations (4) and (5) indicate that the variation in the phase of light is proportional to the line integration of sound pressure. This means acoustical quantities can be obtained from measurement of the phase of light. For measuring the phase, there are several optical techniques which end up with different forms of observation: for example, LDV [1, 4, 5, 7, 9] obtains $c_L \partial \phi_p / \partial t$; Schlieren method [8, 11] obtains $c_S K (\nabla \phi_p)$; and phase-shifting interferometry (PSI) [13] obtains ϕ_p , where c_L and c_S represent system-dependent proportionality constants, and K represents the effect of knife-edge which changes a vector quantity to a scalar.

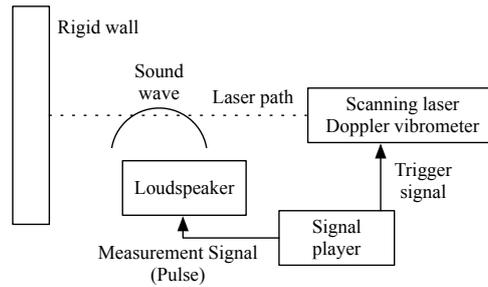


Figure 2: Sound field measurement system by LDV. Time synchronization between LDV measurement and reproduction of sound field is realized by using a trigger signal.

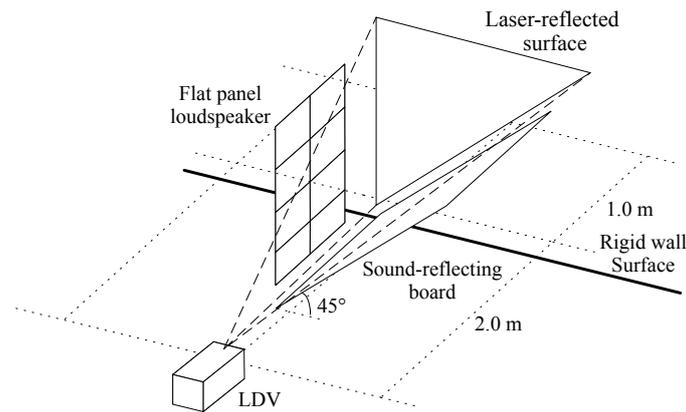


Figure 3: Configuration for observation of reflection of pulse sound wave.

3. SOUND FIELD MEASUREMENT USING SCANNING LASER DOPPLER VIBROMETER [3]

In this section, we describe the method of sound field imaging by scanning LDV and its example based on the acousto-optic effect described in the previous section. In addition, we describe an example of reconstruction of sound field information at each point with computed tomography (CT) in Sec.3.3.

3.1 Method

LDV is normally used to measure the velocity of a vibrating object. For simplicity, we take the Mickelson interferometer as an example to explain the sound field measurement principle of LDV.

Firstly, the laser light is divided into two by a beamsplitter. When one of the laser lights is emitted onto the vibrating object, the frequency or phase of laser light is modulated by the Doppler effect which is caused by change in the light path length. The vibrating object's velocity is obtained by demodulating the Doppler effect using phase difference between the light reflected by the vibrating object and the other light.

As shown in Fig.2, when laser light is emitted onto the object which can be assumed not to vibrate, such as a rigid wall, the LDV can detect changes in the air density along the laser path. Equation (5) is used for the transformation from the obtained signal of phase modulation into sound pressure. Therefore, the information of measured sound field is the integrated information of sound field along the laser path, that is projection of the sound field. Since the diameter of the laser light of LDV is very small, the laser path is assumed to be just a line.

In order to image sound field, it is necessary to measure the sound field along multiple laser paths. Therefore, we restrict the measured object to sound field reproduced by the loudspeaker. Sound field imaging is realized by reproduction of sound field with synchronized-scanning the laser. In the next sections, we show the examples of sound field imaging with LDV.

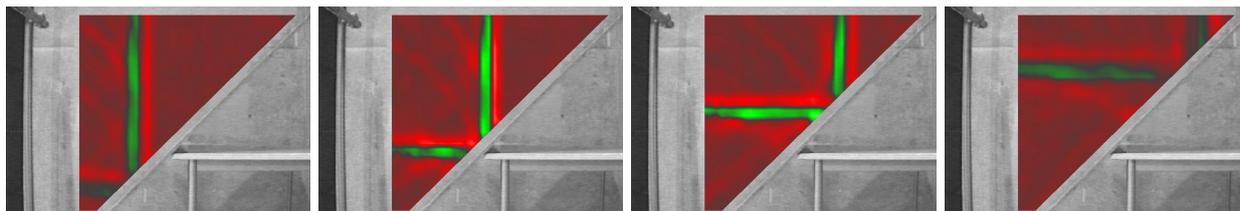


Figure 4: Two-dimensional sound field projections of reflection of pulse sound wave with LDV. Sound field projections obtained in 2.0 ms, 2.5 ms, 3.0 ms and 3.5 ms in left-right order.

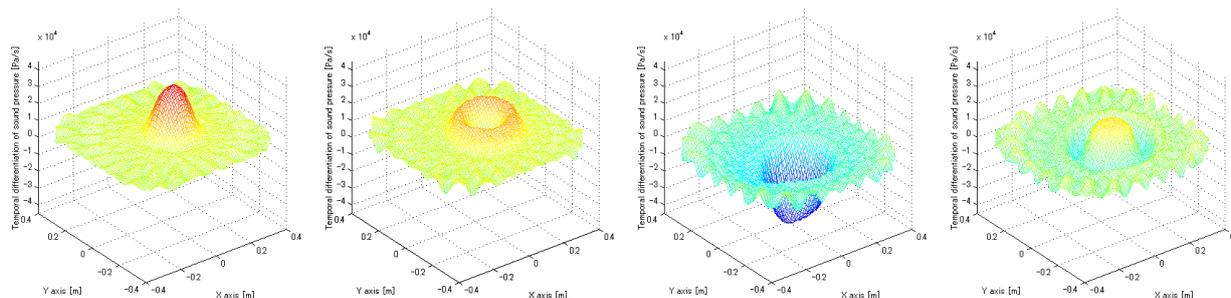


Figure 5: Observation of the plane surface at 0.3 m distance from the front surface of the loudspeaker. Sound information reconstructed in 1.02 ms, 1.09 ms, 1.17 ms and 1.25 ms in left-right order.

3.2 Example of sound field imaging by scanning LDV

We conduct an experiment with a two-dimensional laser projection to observe a reflection, off a sound-reflecting board, of a pulse sound wave propagating from a flat-panel loudspeaker. Figure 3 shows an arrangement of flat-panel loudspeakers and the sound-reflecting board. One cycle of a 4 kHz sine signal is used as the input pulse signal. The sound-reflecting board is a chip board with a thickness of 12 mm and a size of 1820 mm by 910 mm. The output sound pressure level is 100 dB at 1 m from the loudspeaker with the 4 kHz sine signal.

Figure 4 shows the result of the sound field imaging by a LDV (Polytec PSV-300). Since the sound-reflecting board is tilted at a 45-degree angle, we observed in Fig. 4 that the flat sound wave propagated from the flat-panel loudspeaker is reflected, followed by a 90-degree change in the direction of sound propagation. Then, we continue to observe that the sound wave reflected by the ceiling of the room is again reflected by the sound-reflecting board as this experiment is conducted in a closed space. Note that the observed information is a projection of the sound field which is different from the cross-sectional surface of the sound field.

3.3 Example of sound field imaging with computed tomography

We conduct an experiment on the reconstruction of sound field information at each point for the direct sound generated by a 2-way loudspeaker system (YAMAHA NS-10M). we reconstruct the sound field information in the plane at the distance of 30 cm above the surface of the 2-way loudspeaker that is installed facing upward. CT method requires measurements from every direction around the loudspeaker. Since the object of measurement is the direct sound generated by the loudspeaker, we obtain the projection data by rotating the loudspeaker and not the measurement system. The loudspeaker's angle of rotation is 360 degrees in 15-degree increments; the number of measurement points for each projection is 101. As described in the previous section, we use one period of a 4 kHz sine signal as a measurement signal. In this example, the fan-beam CT method is used for the reconstruction of two-dimensional sound field information [2, 3].

Figure 5 shows distributions of the temporal differential of sound pressures calculated from the relationship between the refractive index and sound pressure(Eq.2); the reconstruction of the distribution of refractive indexes is calculated from the differential of the light path length measured using the LDV.

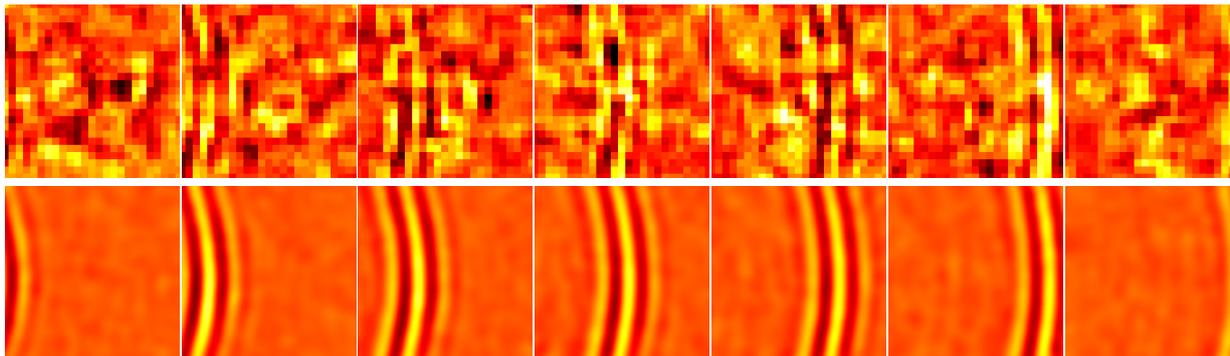


Figure 6: An example of a denoising result by the proposed method applied to measured data obtained by LDV. Pulse emitted from a loudspeaker placed at the left was generated by multiplying the Hann window to 4 period of 4 kHz sinusoidal wave. The upper row shows the raw data, while the bottom row shows the processed result.

4. PHYSICAL-MODEL BASED SIGNAL PROCESSING FOR OPTICAL SOUND MEASUREMENT

As mentioned in Fig. 1 (a), order of magnitude on refractive index variation becomes smaller when sound pressure becomes small. Moreover, as indicated in Sec. 2., some optical measurement methods obtain differentiated information of phase of light that makes measuring quantity related to low-frequency sound smaller. Therefore, especially for an audible sound field, quantity to be measured can be very small that requires a highly sensitive measurement system. Thus, noise contamination can be a problem for optical sound measurement. In addition, physical phenomena other than sound are also able to change refractive index including thermal fluids.

We have proposed some signal processing methods for removing noise contained in data obtained by optical sound measurement [5,7,9,14]. Figure 6 shows an example of effect of the proposed signal processing method applied to measured data obtained by LDV. The pulse emitted from a loudspeaker placed at the left is not easy to recognize from the measured data shown in the top row. On the other hand, visibility of it is greatly enhanced by the proposed method shown in the bottom row.

Basic principle of the proposed method is to utilize the specific property of the scalar wave equation. Since sound wave can be modeled well by the wave equation, observed data of a sound field can be treated as a data obtained from a noise contamination process on a solution of the wave equation. Thus, one can formulate the denoising problem as a recovery problem of a solution to the wave equation from its noisy samples. The Kirchhoff–Helmholtz equation is used to characterize the solutions, and convex optimization techniques are utilized to solve the estimation problem. Such physical-model based characterization allows to reduce not only random noise but also deterministic noise which does not obey the wave equation such as thermal fluid.

5. SINGLE-SHOT MEASUREMENT OF 2D SOUND FIELD USING PARALLEL PHASE-SHIFTING INTERFEROMETRY

In this section, we describe the method of sound field measurement by PPSI and its experimental examples. The advantages of PPSI includes its contactless and quantitative nature. In addition, using a high-speed polarization camera enables to detect two-dimensional (2D) sound field by single-shot. This feature allows us to not only reduce measurement time but also measure an instantaneous field caused by non-reproducible sound sources.

5.1 Method

PPSI is an optical phase measurement method which is derived from PSI. PSI is a well-known optical method for measuring 2D phase distribution of light [31]. It is widely used for measurement of three-dimensional shape. PSI can acquire the 2D phase distribution using several phase-shifted interferometric

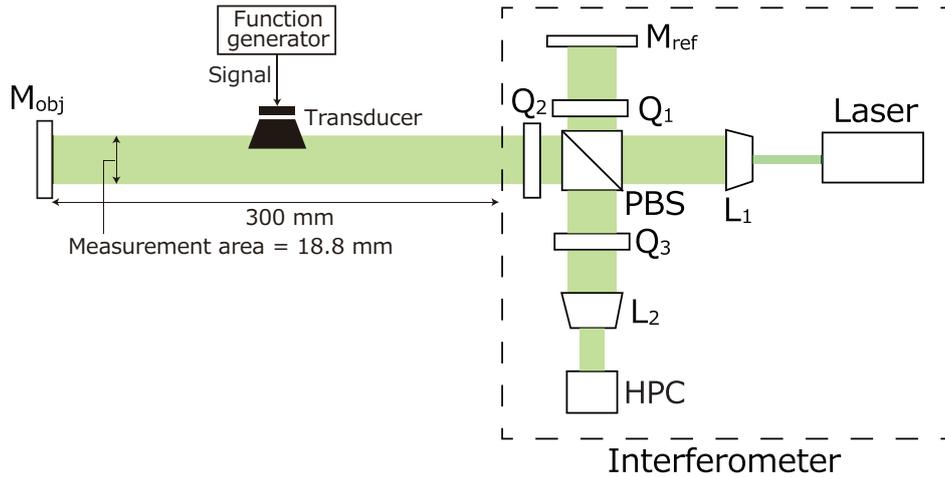


Figure 7: Experimental setup. M: Mirror, L: Lens system, Q: Quarter wave plate, HPC: high-speed polarization camera, PBS: Polarized beam splitter.

images. For example, the four-step PSI can reconstruct the phase using four interferometric images, that can be represented by

$$\phi = \tan^{-1} \frac{I(\frac{3\pi}{2}) - I(\frac{\pi}{2})}{I(0) - I(\pi)}, \quad (6)$$

where $I(\theta)$ is the intensity of interferometric fringe. That is given by

$$I(\theta) = I_0 + I_1 \cos(\phi(x, y) + \theta), \quad (7)$$

where I_0 is the bias intensity, I_1 is the modulation amplitude, ϕ is the 2D phase distribution of light, and θ is the phase retardation of object and reference lights. The phase-shifted images are commonly obtained separately by adjusting the position of optical components; thus, for four-step PSI, four times of measurements are required.

For applying PSI to a moving object or unsteady physical phenomenon, single-shot measurement of multiple phase-shifted images has widely been researched [32–37]. Awatsui *et al.* proposed PPSI which uses a phase-shifting array device for obtaining four phase-shifted images in a single image [35]. Miller *et al.* developed the polarization camera and applied it to PPSI [36, 37]. The polarization camera consists of a phase-shifting array device and an image sensor. The phase-shifting array device consists of four types of linear polarizers whose azimuth are $0, \pi/4, \pi/2$, and $3\pi/4$, respectively. The combination of the polarization interferometer [38] and the polarization camera yields the four phase-shifted images in a single image as shown in Eq. (6). Therefore, PPSI can measure the 2D phase distribution of light by single-shot.

5.2 Example of sound field imaging by PPSI [13]

We conducted a measurement of 40 kHz sound wave using PPSI. The experimental setup are shown in Fig. 7. The high-speed polarization camera (CRYSTA PI-1P made by Photoron Ltd.) was used. The framerate of the camera was 100 kfps, and the active number of pixels was 96×160 ; the equivalent measurement cross-section in the sound field was $18.8 \text{ mm} \times 31.3 \text{ mm}$. The laser with a wavelength of 532 nm and power of 70 mW was used. The ultrasonic transducer (MURATA MA40S4S) was used for generating a sound field.

Figures 8 shows experimental results when one transducer was driven by pure tone of 40 kHz. It can be seen that the instantaneous sound fields are captured. The value of the results are depicted in units of length; that is calculated by the phase of light multiplied by the wavelength of light. The acoustical quantities are derived using Eqs. (4) and (5). Figure 9 plots experimental results when two transducers were driven by the same pure tone of 40 kHz. The interference patterns of sounds can be shown in the figures. Those results indicate that the PPSI is effective for high-speed and instantaneous measurement of a 2D sound field.

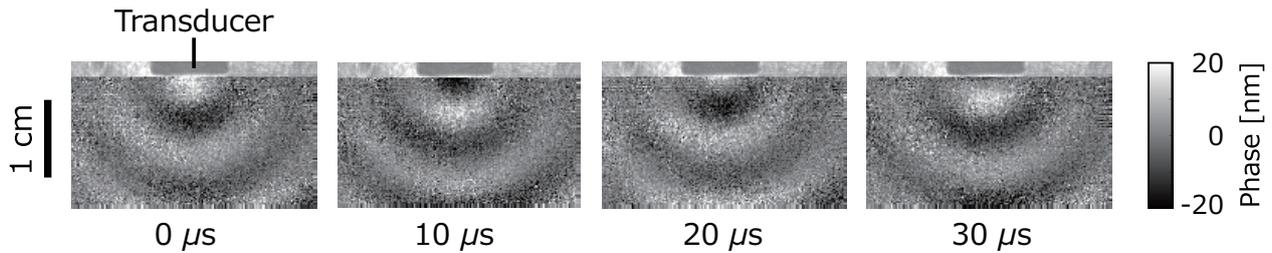


Figure 8: Obtained instantaneous phase distribution of light that travels through the sound field generated by one ultrasonic transducer which is driven by pure tone of 40 kHz. The time interval of the images are 10 μ s.

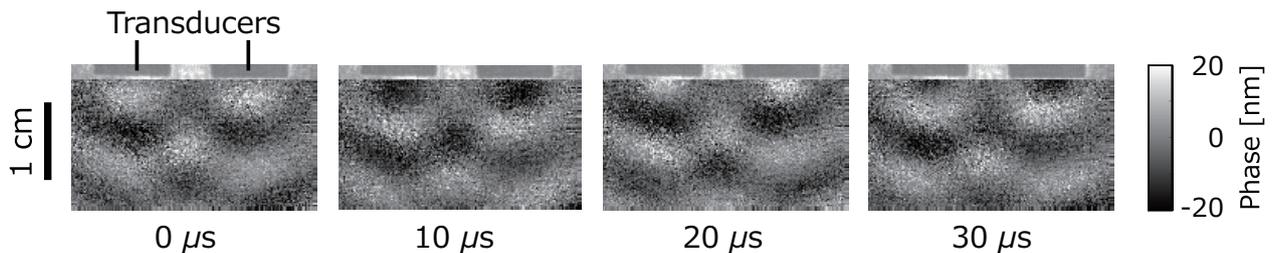


Figure 9: Obtained instantaneous phase distribution of light that travels through the sound field generated by two ultrasonic transducers which are driven by the same pure tone of 40 kHz. The time interval of the images are 10 μ s.

6. CONCLUSIONS

In this paper, we gave an outline about acousto-optic effect, which is sound effect on the light. The sound field measurement and imaging method using LDV was shown. Some signal processing techniques such as CT and physical-model based optimization techniques allows the extraction of sound information from the measurement data. We also demonstrated the acoustical measurement and imaging using PPSI with a high-speed polarization camera. It is the most advanced optical measurement system, and has many applications. It is useful to measure the sound field which is close to the sound source or the sound field in the very small space.

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