Numerical analysis of acousto-optic effect caused by audible sound based on geometrical optics

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Paper Number: O6000218

Abstract

The acousto-optic effect is well recognized as optical wavefront modulation caused by sound wave in the field of ultrasonics and optics. In contrast, the effect caused by audible sound has not been studied sufficiently because there were few applications. However, in recent years optical sound measurement techniques based on the effect has attracted attention. This paper reports numerical analysis of the acousto-optic effect by the audible sound: deflection and phase modulation. According to our analysis, the deflection of an optical ray in an audible sound field can be ignored. Variation in a phase is at most several nanometer when light propagates 10 times of the wavelength of sound in a sound field whose sound pressure is 1 Pa. It is also confirmed that the acousto-optic effect becomes larger as a propagation direction of light and sound becomes perpendicular; therefore, the effect has a directivity to a plane wave.

***Index Terms*— Light, laser Doppler, optical sound measurement, deflection, eikonal, inhomogeneous media.**

1. Introduction

Light propagating in a sound field is affected by variation in the refractive index of air because the refractive index is changed by the sound. That effect is called acousto-optic effect. The interaction between ultrasound and light is formulated as Raman–Nath diffraction [1] and Bragg diffraction [2]; the effect has been applied to optical devices which control an optical wavefront. On the other hand, optical sound measurements based on the effect caused by audible sound has been studied. There are several methods, for example, laser Doppler vibrometer (LDV) [3], optical wave microphone [4], schlieren method [5], and digital holography [6]. However, a fundamental theory for the acousto-optic effect caused by audible sound has not been established. Raman–Nath and Bragg formulations cannot be applied to the effect caused by audible sound because those formulations assume that several periods of a sound wave exist in an incident optical beam. In addition, since those methods had been developed in different fields and applied to sound measurements, theories and models of each method are different despite the same physical phenomenon. For these reasons, constructing a fundamental theory that can be applied to the entire optical sound measurement methods is required.

In this paper, the equation of sound pressure and refractive index of air was derived from an assumption of adiabatic change and the Gladstone–Dale relation. The theory of light in an audible sound field was established, and the acousto-optic effect was analyzed numerically.

2. Relation between sound pressure and refractive index

Propagation of light is characterized by the refractive index of a medium. Considering the propagation of light in air, the refractive index and density of air is governed by the Gladstone–Dale relation:

$$\frac{n-1}{\rho} = \text{const.}, \tag{1}$$

where n is the refractive index of air, and ρ is the density of air [7]. Process of density variation caused by sound can be assumed as adiabatic change; the relation between the density of air and sound pressure is

$$\frac{p_0 + p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^{\frac{1}{\gamma}}, \tag{2}$$

where p_0 and ρ_0 are the pressure and the density under static conditions, p is the sound pressure, and γ is the specific heat ratio. According to Eq. (1) and Eq. (2), the relation of the refractive index of air and the sound pressure becomes

$$n = (n_0 - 1)\left(1 + \frac{p}{p_0}\right)^{\frac{1}{\gamma}} + 1, \tag{3}$$

where n_0 is the refractive index of air under static conditions. This equation reveals that the relation between the refractive index and the sound pressure is nonlinear. By performing the Taylor expansion with respect to p, Eq. (3) becomes

$$n = n_0 + \frac{n_0 - 1}{\gamma p_0} p - \frac{(\gamma - 1)(n_0 - 1)}{2\gamma^2 p_0^2} p^2 + \ldots . \tag{4}$$

12th Western Pacific Acoustics Conference 2015. Edited by Lim Kian Meng

ISBN: 978-981-09-7961-4 — doi: 10.3850/978-981-09-7961-4_O6000218

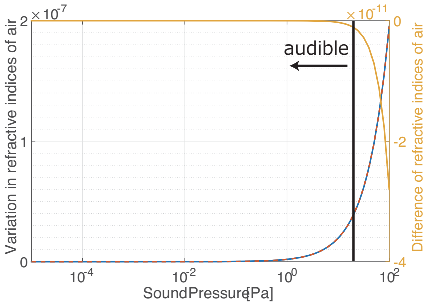

Fig. 1. The left axis shows the variation in the refractive indices of air from the static value calculated by Eq. (3) (blue line) and Eq. (6)(dashed red line). The right axis shows the difference of the two lines (orange line). The constants are the following: $n_0 = 1.000279$, $\gamma = 1.41$, and $p_0 = 101325$.

Substituting constants in the equation: n_0, γ, p_0 by 1.000279, 1.41, 101325, respectively; the refractive index of air is given by

$$n = 1.000279 + 1.9668\times10^{-9}p - 2.7730\times10^{-15}p^2 + \ldots \tag{5}$$

Let Eq. (4) be approximated in the first order of the sound pressure, the refractive index becomes

$$n \simeq n_0 + \frac{n_0 - 1}{\gamma p_0}p. \tag{6}$$

The refractive indices of air calculated by Eq. (3) and Eq. (6) are plotted in Fig. 1, the difference of the two curves is also plotted in the figure. By assuming the upper limit of audible sound pressure is 20 Pa, the error of refractive index for the audible sound due to the first order approximation is less than 10^{-12}. Here it can be seen that the first order approximation has sufficient precision for the audible sound field.

3. Light propagation in an audible sound field based on geometrical optics

Since sound causes a change in the refractive index of air, light propagating in sound field can be formulated as optical phenomena in inhomogeneous media. The light propagating in inhomogeneous media, whose oscillation frequency is much smaller than that of light, is given by the wave equation:

$$\nabla^2 U - \frac{n^2}{c^2}\frac{\partial^2 U}{\partial t^2} = 0, \tag{7}$$

where U is scalar representation of an electric field, and c is speed of light [8]. A solution of the wave equation is assumed to be of the form

$$U_\omega(\boldsymbol{r}, t) = A(\boldsymbol{r})\mathrm{e}^{ik(L(\boldsymbol{r})-\omega t)}, \tag{8}$$

where U_ω is the scalar electric field for ω, $\boldsymbol{r} = (x, y, z)$, k is the wavenumber of light, ω is the angular frequency of light, A is the amplitude, and L is the eikonal that represents wavefront of light. By substituting Eq. (8) back into Eq. (7), the real term of the equation becomes

$$n^2 - |\nabla L|^2 + \frac{\nabla^2 A}{k^2 A} = 0. \tag{9}$$

Since a modulation period of the amplitude by sound is on the same order of that of the sound, the third term of Eq. (9) can be ignored. Thus, Eq. (9) is approximately

$$n^2 = |\nabla L|^2, \tag{10}$$

that is called eikonal equation which is the fundamental equation of geometrical optics. The integral form of the eikonal equation is

$$L = \int_C n ds = \int_C \left(n_0 + \frac{n_0 - 1}{\gamma p_0}p\right) ds, \tag{11}$$

where s is the optical length. The integration is conducted along the optical path, C. While the phase of light can be calculated by using this equation, the optical path is still unknown. In general cases, the optical path cannot be obtained analytically; solving a differential equation is required.

4. Calculation of deflection of ray caused by an audible sound field

An optical ray is deflected by sound and that is described by a differential equation. Ray equation [9], which describes a path of a ray, is given by

$$\frac{d}{ds}\left(n\frac{d\boldsymbol{r}}{ds}\right) = \nabla n. \tag{12}$$

By setting initial values to Eq. (12), the optical path is determined.

4.1. Analytical solution for plane wave

For simplicity, let us consider a two-dimensional field. The path of the ray when the refractive index varies only in x-direction is described by

$$z = \int \frac{n(x_0)\cos\phi_0}{\sqrt{[n(x)]^2 - [n(x_0)\cos\phi_0]^2}} dx, \tag{13}$$

where x_0 is the initial position in x-axis and ϕ_0 is the initial angle between the ray and z-axis [9]. The definition of the coordinates is shown in Fig. 2. Figure 3

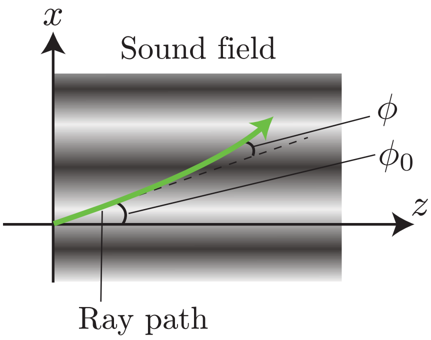

Fig. 2. Definition of the coordinates. The x-axis is corresponding to the direction of the sound propagation; ϕ_0 is the initial angle between the ray and z-axis; and ϕ is the deflection angle. The green curve illustrates the ray path and the gray tint illustrates the variation of the refractive index.

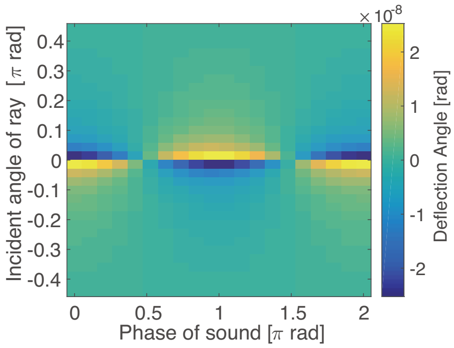

Fig. 3. Deflection angle ϕ when the peak to zero value of the sound pressure is 1 [Pa]; the frequency of the sound is 1000 [Hz]; propagation distance of the ray is 10Λ; Λ is the wavelength of the sound. The horizontal axis is the phase of the sound, and the vertical axis is the incident angle of the ray. The initial position of the ray is the origin. The sound pressure $p(x)$ is given by $p(x) = \cos(Kx + \psi)$, where K is the wavenumber of the sound, ψ is the phase of sound; and the refractive index is given by Eq. (6), where $n_0 = 1.000279$, $\gamma = 1.41$, $p_0 = 101325$.

shows the deflection angle ϕ as a function of phase and direction of the plane sound wave. The deflection is large when the propagation direction of light and sound is nearly perpendicular. This is because the deflection is canceled out when the ray propagates one period of the sound wave. According to the Fig. 3, the deflection angle was of order 10^{-8}. Considering the size of a photo-electric sensor, the deflection of light caused by the audible sound can be ignored.

4.2. *Analytical solution for spherical wave*

Considering two-dimensional polar coordinates (r, θ), and setting the center of a spherical sinusoidal sound wave at the origin of the coordinates. Since the refractive index varies only in r direction, the path of the ray is given by

$$\theta = \int \frac{r_0 n(r_0) \sin\gamma_0}{r\sqrt{[rn(r)]^2 - [r_0 n(r_0)\sin\phi_0]^2}} dr, \qquad (14)$$

where r_0 is the initial value of r-axis, ϕ_0 is the angle illustrated in Fig. 4 [9]. Figure 5 shows the deflection angle as a function of the phase of the sound and the initial direction of the ray. The deflection angle becomes larger as the direction of the sound and the ray approaches perpendicular, as with the case of the plane wave. The deflection angle is of order 10^{-9}. The deflection by the spherical wave was smaller than that by the plane wave in spite of the same propagation distance and sound pressure. This is because the direction of the deflection becomes opposite when the phase of the sound field becomes inverse. Therefore, the deflection angle becomes maximum when the direction of a ray is perpendicular to that of a plane sound wave. According to Fig. 3, since the deflection angle was up to 3×10^{-8} when the ray propagates 10 times of the wavelength of 1 Pa plane sound wave, the deflection of the ray in the audible sound field can be ignored; the ray advances straight.

5. Calculation of the acousto-optic effect

As we discussed in the previous section, deflection of ray due to an audible sound field can be ignored. By applying the approximation, the eikonal can be written as

$$L = n_0 Z + \frac{n_0 - 1}{\gamma p_0} \int p dz, \qquad (15)$$

where Z is the length of the optical path and z-axis is direction of a ray. The first term of Eq. (15) is the phase rotation due to propagation in a homogeneous medium. Thus, phase modulation caused by the sound can be defined as

$$L_s = \frac{n_0 - 1}{\gamma p_0} \int p dz. \qquad (16)$$

The unit of L_s is length and variation in a phase of light is derived by multiplying the wave number of the

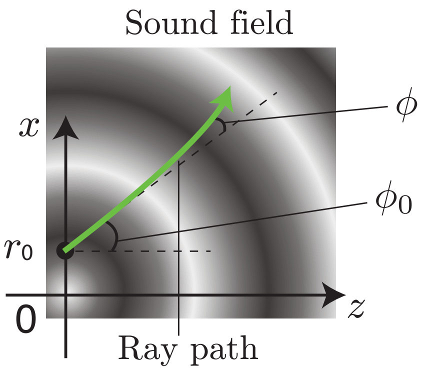

Fig. 4. Definition of the coordinates. The center of the spherical sound wave is at the origin of the coordinates. The symbol r_0 is the initial value of r-axis, ϕ_0 is the initial angle of the ray between the ray and the tangent of a circle at the initial position, ϕ is the deflection angle of the ray.

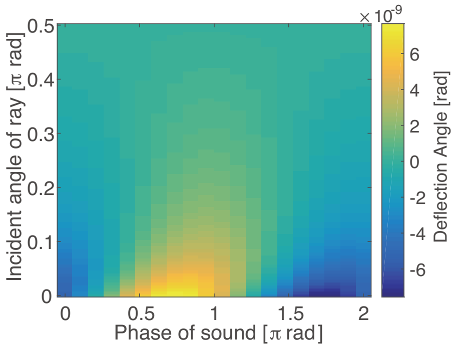

Fig. 5. Deflection angle ϕ when the peak to zero value of the sound pressure is 1 [Pa]; the frequency of the sound is 1000 [Hz]; $r_0 = 5\Lambda$; propagation distance of the ray is 10Λ; Λ is the wavelength of the sound. The horizontal axis is the phase of the sound, the vertical axis is the incident angle of ray, ϕ_0, described in Fig. 4. The sound pressure $p(r)$ is given by $p(r) = 1/r \cos(Kr + \psi)$, where K is the wavenumber of the sound, ψ is the phase of sound; and the refractive index is given by Eq. (6), where the constants in Eq. (6) are the same as Fig. 3.

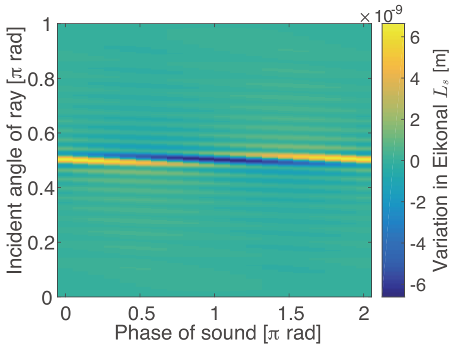

Fig. 6. The variation in the eikonal calculated by Eq. (16) when the peak to zero value of the sound pressure is 1 [Pa]; the frequency of the sound is 1000 [Hz]; the propagation distance is 10 times of wavelength of the sound. The horizontal axis is the phase of the sound and the vertical axis is the incident angle between the propagation direction of the sound and that of light. The constants in Eq. (16) are the same as Fig. 3.

light to L_s. Since a wave number depends on a light source, variation in a phase also depends on that. In the following discussion, L_s is used for representing the acousto-optic effect.

Figure 6 shows L_s of the modulated light. The figure indicates that the variation in the eikonal was on the order of 1 nm. Since a wavelength of visible light was on the order of 100 nm, the phase of the light was shifted about 1/100 period. Detecting this phase shift enables to measure integrated sound pressure. LDV and holography detect the phase modulation by using optical interference techniques [3,6]. Figure 7 plots the maximum value of L_s in Fig. 6 for each incident angle. The phase modulation becomes maximum when the propagation direction of the sound is perpendicular to that of the light. The variation in the eikonal becomes zero when the ray propagates through the integer multiple of the period of the sound field, because the eikonal takes opposite sign when the sign of the sound pressure becomes opposite. The figure indicates that measurements of the acousto-optic effect have a directivity. Since the integration of the field is continuous, the spatial aliasing due to discretely arranged measurement instruments such as a microphone array does not appear on the acousto-optic measurements. This advantage is utilized for an acousto-optic beamformer and near-field acoustic holography [10, 11].

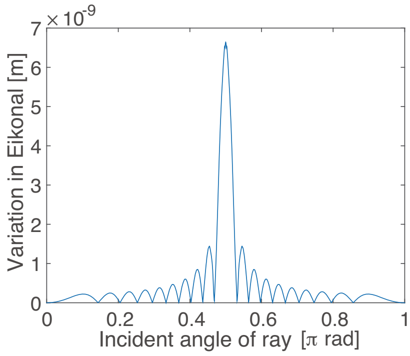

Fig. 7. The maximum value of the variation in the eikonal depicted in Fig. 6 as a function of the incident angle.

6. Conclusions

This paper presented the derivation and the numerical analysis of the acousto-optic effect caused by an airborne audible sound field. By considering sound in only audible range, the formulation of the acousto-optic effect was established, which was different from that in ultrasonic range. The calculation of the deflection angle indicated that the deflection could be ignored; that provided the validity of the straight-propagation approximation. Consequently, calculation of the acousto-optic effect for the arbitrary audible sound field became possible. The directivity of the acousto-optic effect to a plane wave was also confirmed by the numerical analysis.

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